EFFECT OF VOLUME FRACTION ON NON-NEWTONIAN FLUID THROUGH A CHANNEL WITH POROUS MEDIUM

*Monika Srivastava[1], Rajeev Khare[2]

Department of Mathematics & Statistics, Sam Higginbottom University of Agriculture, Technology And Sciences
Allahabad Uttar Pradesh
monikasrivastava945@gmail.com

Abstract- *The effect of magnetic field on the flow of non-Newtonian fluids through different channels containing porous medium has very important role for industries in designing manufacturing metallic items. In the present paper a study on the effect of volume fraction on the flow of a non-Newtonian fluid through vertical channel filled with porous medium under the transversely applied magnetic field. And after forming the governing equations of the flow and establishing the boundary conditions according to the flow nature. The result clearly indicates that the velocity profile of fluid can be easily controlled by applying magnetic field.*
Keywords: Hall Current; Volume fraction, Visco-elastic flow; Porous Medium; Thermal Radiation

1 Introduction

The study of non-Newtonian fluid flows has gained the attention of engineers and scientist in present times due to its important applications in various branches of sciences, engineering and technology Magnet-hydrodynamic flow of fluid has a very important in science and technology : particularly in chemical and nuclear industries, practically, the flow occurs through porous medium and the fluid existing in nature are generally non-Newtonian therefore this study concentrates the effect of volume fraction on non-Newtonian fluid flowing in a magnetic field through vertical plate channel completely filled with porous medium.

The objective of this paper is to find the effect of volume fraction in the presence of magnetic field on velocity profile of the fluid flowing through a porous channel. Several researchers contributed their valuable work in this field. Suzuki and Tanka performed some experiments on non-Newtonian fluid along an inclined plane .Nigam and Singh [1960]; Sondalgekar and Bhat [1971];Attia and Kob,[1996] studied the effect of transversely applied magnetic field on convection flows of an electrically conducting fluid. Rapits et al., [1982] studied hydro-magnetic free convection flow through porous medium between two parallel plates. Singh, Garg and Bansal [2014] studied Hall current effect in visco-elastic MHD oscillatory convective flow through a porous medium in a vertical channel with heat radiation. Since most of the fluid occurring in nature or non-Newtonian and they are of great industrial important so in the present paper a studied effect of a volume fraction on non-Newtonian fluid in magnetic field through channel having porous medium has been done.

Formulation of the problem:

Let us consider a non-Newtonian incompressible fluid through an electrically conducting porous medium existing between two infinite vertical plates in the presence of volume fraction, hall currents and thermal radiation. The two plates are considered at a distance d apart. Considering the Cartesian co-ordinate system with x'-axis fixed vertically upward along the centre line of the channel. The z' –axis is assumed perpendicular to the planes of the plates along which a strong transverse magnetic field is applied.

In this paper, non-Newtonian factor has been introduced with magnetic field term to study the behaviour of fluid velocity. All the physical quantities except the pressure depend only on z' and t' only. Let (u', v', w') be the components of velocity in directions (x', y', z') respectively. Since the plates are non-porous, therefore equations of continuity ($\nabla \cdot V = 0$) on integration gives $w' = 0$.

$$div\overline{\mathrm{B}} = 0$$

$$\overline{\mathrm{B}} = \left(B_x', B_y', B_z'\right), B_z' = B_\circ \quad \text{(constant).}$$

Assumed no applied and polarization voltage exists. $\overline{E} = 0$.

$$\bar{j} + \frac{\omega_e \tau_e}{\mathrm{B}_\circ}(J \times \mathrm{B}) = \sigma(V \times \mathrm{B}) \text{ (Ohm's law)} \qquad (1)$$

the conservation of electric charge div $\bar{j}$ =0

$$\text{gives}\left(j_z' = 0\right) \text{ (constant)} \qquad (2)$$

$J_z' = 0$ at the plates and hence zero everywhere in the fluid. Under the assumptions that the electron pressure (for weakly ionized gas) the thermo electric pressure, ion slip and the external electric field arising due to polarization of charges is negligible

Equation (1) becomes,

$$j_x' + \omega_e \tau_e j_y' = \sigma B_\circ \bar{v}' \quad (3)$$

$$j_y' + \omega_e \tau_e j_x' = \sigma B_\circ \bar{u}' \quad (4)$$

Solving equation (3) and (4) for j_x' *and* j_y'

$$j_x' = \frac{\sigma B_\circ}{(1+H^2)}(Hu' + v') \quad (5)$$

And

$$j_y' = \frac{\sigma B_\circ}{(1+H^2)}(Hv' + u') \quad (6)$$

Where, $H = \omega_e \tau_e$ (Hall parameter)

Under the Boussinesq approximation momentum equation in Cartesian components reduce to

$$(1-\phi)\frac{\partial u'}{\partial t'} = -\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \nu_1 \frac{\partial^2 u'}{\partial z'^2} + \nu_2 \frac{\partial^3 u'}{\partial z'^2 \partial t'} + \frac{\sigma B_\circ^2 (Hv' - u')}{\rho(1+H^2)(1+c^2)} - \frac{\nu_1 u'}{K'} + g\beta(T' - T_\circ), \quad (7)$$

$$(1-\phi)\frac{\partial v'}{\partial t'} = -\frac{1}{\rho}\frac{\partial p'}{\partial y'} + \nu_1 \frac{\partial^2 v'}{\partial z'^2} + \nu_2 \frac{\partial^3 v'}{\partial z'^2 \partial t'} + \frac{\sigma B_\circ^2 (Hu' - v')}{\rho(1+H^2)(1+c^2)} - \frac{\nu_1 v'}{K'}, \quad (8)$$

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial z'^2} - \frac{\partial q'}{\partial z'} \quad (9)$$

Corresponding boundary conditions are:

$$u' = v' = 0 \;;\; T' = T_\circ \text{ at } z' = -\frac{d}{2}, \quad (10)$$

$$u' = v' = 0,$$

$$T' = T_\circ + (T_w - T_\circ)\cos\omega' t' \quad \text{at} \quad z' = -\frac{d}{2}, \quad (11)$$

$$\frac{\partial q'}{\partial z'} = 4\alpha^2 (T' - T_\circ) \quad (12)$$

Stand for radiative heat flux,

$$\alpha^2 = \int_0^\infty k_{\lambda n} \frac{\partial e_{b\lambda}}{\partial T} d\lambda. \quad (13)$$

Where $k_{\lambda n}$: absorption coefficient at the walls.

$e_{b\lambda}$: Plank's function.

Non-dimensional quantities are:

$$\eta = \frac{z'}{d}, x = \frac{x'}{d}, y = \frac{y'}{d}, u = \frac{u'}{U}, v = \frac{v'}{U}, T = \frac{T' - T_\circ}{T_w - T_\circ}$$

$$t = \frac{t'U}{d}, \omega = \frac{\omega' d}{U}, p = \frac{p'}{\rho U^2}$$

Using non-dimensional quantities and solving equations (7) to (9) and using equation (11),

$$R_e(1-\phi)\frac{\partial u}{\partial t} = -R_e \frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial \eta^2} + \gamma \frac{\partial^3 u}{\partial \eta^2 \partial t} + \frac{M^2(Hv - u)}{(1+H^2)(1+c^2)} - \frac{1}{K}u + G_r T. \quad (14)$$

$$R_e(1-\phi)\frac{\partial v}{\partial t} = -R_e \frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial \eta^2} + \gamma \frac{\partial^3 v}{\partial \eta^2 \partial t} + \frac{M^2(Hu - v)}{(1+H^2)(1+c^2)} - \frac{1}{K}v. \quad (15)$$

$$P_e \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial \eta^2} - N^2 T. \quad (16)$$

The corresponding transformed boundary conditions are:

$$u = v = 0, T = 0 \quad \text{at} \quad \eta = -\frac{1}{2} \quad (17)$$

$$u = v = 0, T = \cos\omega t \quad \text{at} \quad \eta = \frac{1}{2} \quad (18)$$

Where,

$R_e = \dfrac{Ud}{\nu_1}$: Reynold's number,

$K = \dfrac{K'}{d^2}$: The permeability of porous medium.

$M = B_\circ d\sqrt{\dfrac{\sigma}{\rho\nu_1}}$: Hartmann number,

$\gamma = \dfrac{\nu_2 R_e}{d^2}$: Visco-elastic parameter,

Φ : Volume fraction,

$H = \omega_e \tau_e$: Hall parameter,

$G_r = \dfrac{g\beta d^2 (T_W - T_\circ)}{\nu_1 U}$: Grashof number,

$N = \dfrac{2\alpha d}{\sqrt{k}}$: Radiation parameter,

$P_e = \dfrac{\rho C_p dU}{k}$: Peclet number,

Following Singh and Pathak [2013], for the oscillatory internal flow considered we shall assumed the fluid flows only under the influence of a non-dimensional pressure gradient oscillating in the direction of x-axis which is the form

$$-\frac{\partial p}{\partial x} = A\cos\omega t \quad \text{and} \quad -\frac{\partial p}{\partial y} = 0 \tag{19}$$

Solution of the problem:

Combining equations (14) and (15), introduce a complex function F=u+iv and using equation (19),

$$R_e \frac{\partial p}{\partial x} - G_r T = \gamma \frac{\partial^3 F}{\partial \eta^2 \partial t} + \frac{\partial^2 F}{\partial \eta^2} - R_e(1-\phi)\frac{\partial F}{\partial t} - \left(\frac{M^2(1+iH)}{(1+H^2)(1+c^2)} + K^{-1}\right)F \tag{20}$$

The boundary conditions (17) and (18) in complex form can be written as:

$$F = 0, T = 0 \quad \text{at} \quad \eta = -\frac{1}{2,} \tag{21}$$

$$F = 0, T = \cos\omega t \quad \text{at} \quad \eta = \frac{1}{2,} \tag{22}$$

Solving equations (16) and (20) using boundary conditions (21) and (22),

Assume complex form the solution of the problem as:

$$F(\eta,t) = F_\circ(\eta)e^{i\omega t}, T(\eta,t) = \theta_\circ(\eta)e^{i\omega t}$$
$$and$$
$$-\frac{\partial p}{\partial x} = Ae^{i\omega t} \tag{23}$$

The boundary conditions (21) and (22) become:

$$F = 0, \theta_\circ = 0 \;\; \eta = -\frac{1}{2,} \tag{24}$$

$$F = 0, \theta_\circ = 1 \;\; \eta = \frac{1}{2,} \tag{25}$$

Putting the value of equation (23) in equations (16) and (20),

$$l^2 F_\circ'' - \Psi^2 F_\circ = -AR_e - G_r \theta_\circ \tag{26}$$

and

$$\theta_\circ'' - n^2 \theta_\circ \tag{27}$$

Where,

$$\Psi^2 = \left\{\frac{M^2(1+iH)}{(1+H^2)(1+c^2)} + K^{-1} + i\omega R_e(1-\phi)\right\}$$

$$l^2 = (1 + i\omega\gamma)$$

$$n^2 = N^2 + i\omega P_e$$

The ordinary differential equation (26) and (27) are solved under the boundary conditions (24) and (25), the solution of the problem is obtained:

$$F = \left[\begin{array}{c} \dfrac{AR_e}{\Psi^2}\left\{1 - \dfrac{\cosh\frac{\Psi}{l}\eta}{\cosh\frac{\Psi}{2l}}\right\} + \dfrac{G_r}{(l^2 n^2 + \Psi^2)} \\ \left\{\dfrac{\sinh\dfrac{\Psi}{l\left(\eta + \frac{1}{2}\right)}}{\sinh\frac{\Psi}{l}} - \dfrac{\sinh n\left(\eta + \frac{1}{2}\right)}{\sinh n}\right\} \end{array}\right] e^{i\omega t} \tag{28}$$

$$T(\eta,t)=\left[\frac{\sinh\left(\eta+\frac{1}{2}\right)}{\sinh n}\right]e^{i\omega t} \quad (29)$$

To observe the velocity profile with respect to applied magnetic field, suitable values for different parameters are chosen to plot the curve between magnetic field and velocity profile for different values of non-Newtonian factors.

Result and Discussion

Table: Magnetic field and Velocity profile.
The values of parameters are:
(H =1, K=0.1, N=1, A= 4, $R_e = 0.5, P_e = 1, G_r = 5$, ϒ=0.1)

	Velocity Profile				
Hartmann Number (H)	Volume Fraction $(\varphi) = 0.1$	Volume Fraction $(\varphi) = 0.5$	Volume Fraction $(\varphi) = 1.0$	Volume Fraction $(\varphi) = 1.5$	Volume Fraction $(\varphi) = 2.0$
0.1	1.222730	1.302345	1.413035	1.538181	1.680469
0.5	1.216670	1.295700	1.405542	1.529689	1.670787
1.0	1.198016	1.275249	1.382502	1.503594	1.641061
1.5	1.167843	1.242204	1.345320	1.461543	1.593234
2.0	1.127433	1.198009	1.295689	1.405531	1.529676
2.5	1.078409	1.144491	1.235736	1.338050	1.453329
3.0	1.022596	1.083694	1.167827	1.261858	1.367427
3.5	0.961887	1.017726	1.094384	1.179752	1.275219
4.0	0.898115	0.948615	1.017719	1.094376	1.179743
4.5	0.832962	0.878207	0.939911	1.008083	1.083670
5.0	0.767890	0.808095	0.862733	0.922851	0.989212
5.5	0.704112	0.739583	0.787618	0.840252	0.898092
6.0	0.642580	0.673682	0.715654	0.761455	0.811565
6.5	0.583997	0.611128	0.647614	0.687268	0.730466

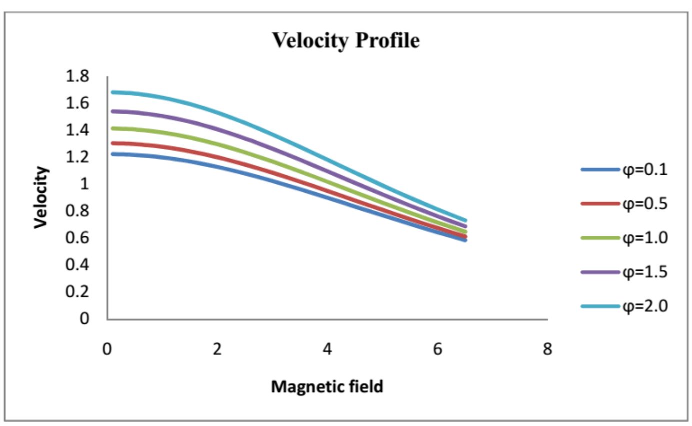

Fig.1

It is indicated from the graph that the velocity decreases with increase of magnetic field for every value of volume fraction, the form of each curve remains the same, initially all start from a fixed value of velocity, in hyperbolic nature, become parallel to the x-axis. However, the curvature is highly affected by the volume fraction. It appears from the derived relation for velocity that volume fraction is very important role for controlling the velocity as it is associated with many terms present in the relation. Also the hyperbolic functions of various parameters play the role in controlling the velocity. The transverse magnetic field on an electrically conducting fluid creates a resistive force which looses the velocity. The presence of other parameters cannot be ignored

Hence motion of such fluids can be controlled easily by magnetic field and volume fraction and this can find applications in metallic industries.

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