Free Vibration Analysis of Symmetric Laminated Composite Plate using Finite Element Method

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ABSTRACT

The vibration analysis of symmetric laminated composite plates is analyzed using finite element method. The natural frequencies and mode shapes are determined for different boundary condition in this analysis. Comparisons are made with the result for thin and thick composite laminated plate. The model has been develop using an appropriate eight node isoparametric element (SHELL281) from the ANSYS element library. In this paper Numerical results have been computed for the effect of different boundary conditions, thickness ratio, and different aspect ratio, number of layers and material properties of laminated composite plate. The effect of thickness ratio is found that as we increase the thickness ratio non dimensional frequency decreases. And the effect of aspect ratio, number of layers and material properties of laminated composite plate is found that non dimensional frequency increases with increase of aspect ratio, number of layers and ratio of material properties.

Keywords: *Free vibration, Boundary condition, Laminate composite plate.*

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INTRODUCTION

A composite is a structural material that consists of two or more combined constituents that are combined at a macroscopic level and are not soluble in each other. One constituent is called the reinforcing phase and the one in which it is embedded is called the matrix. The reinforcing phase material may be in the form of fibers, particles, or include concrete reinforced with steel and epoxy reinforced with graphite fibers, etc. flakes. The matrix phase materials are generally continuous. Examples of composite systems include concrete reinforced with steel and epoxy reinforced with graphite fibers etc. Laminated composite plates are mostly used in many engineering applications such as mechanical, sports, biomedical, aerospace, marine, automobile, heavy machinery, agricultural equipment due to their high strength to weight ratio, low specific density, long fatigue life, high stiffness to weight ratio, low weight, high modulus, good electrical and thermal conductivity and other superior material properties.

There are many publications about symmetrically laminated plate vibration. C.W. Bert et. al. [1] studied free vibration of un-symmetrically laminated anisotropic plate with clamped edges. J.M. Whitney et. Al. [2] studied Analysis of simply supported laminated anisotropic Rectangular plate. E. Reissner [3] studied A consistent treatment of transverse shear deformations in laminated anisotropic plates. Khedeir [5] studied An exact approach to the elastic state of stress of shear deformable anti-symmetric angle ply laminated plates Wang et. al. [6] studied the free vibration analysis of skew fiber reinforced composite laminates based on first-order shear deformation plate theory. J.N. Reddy et. al. [7] studied An exact solution for the bending of thin and thick cross-ply laminated beams. Aydogdu et. al. [9] studied the Vibration analysis of cross ply laminated square plates with general boundary conditions. Ashour [11] studied Vibration of angle-ply symmetric laminated composite plates with edges elastically restrained. Ganapathi et. al. [12] studied Free Vibration analysis of simply supported Laminated Composite panels. Khdeir [13] studied comparison between shear deformable and Kirchhoff theory for bending, buckling, and vibration of anti-symmetric angle ply laminated plates. Sang Wook Kang et. al. studied Vibration analysis of simply supported rectangular plates with unidirectional, arbitrarily varying thickness. Karami et. al. [15] studied DQM free vibration analysis of moderately thick symmetric laminated plates with elastically restrained edges. Sharma et. al. [16-19] studied the free vibration analysis of laminated composite plates considering first order shear deformation theory.

ELEMENT AND METHODOLOGY

SHELL 281 is a eight-node linear shell element with six degrees of freedom at each node. Those are translation in x, y, z direction and rotation about x, y, z axis. It is well-suited for linear, large rotation, and large strain nonlinear applications. The element formulation is based on logarithmic strain and true stress measures. Fig. 1 shows the idea regarding the SHELL281 element. The details of the element can be seen in reference [20].

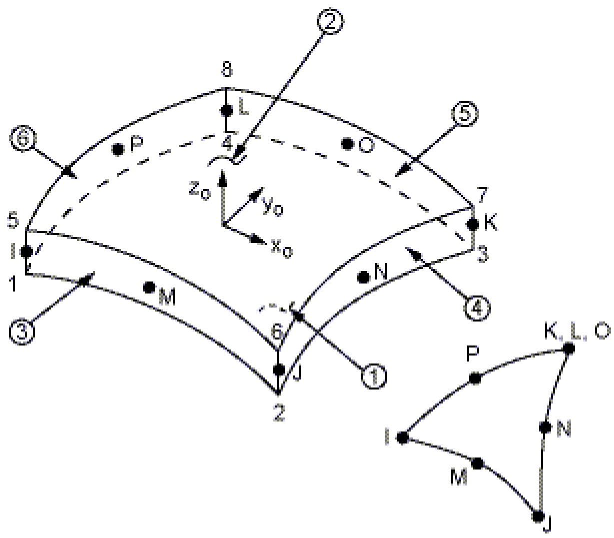

Figure 1 SHELL 281

Subspace Iteration method was used for free vibration analysis of laminated plate. The subspace iteration method is given by Bathe [21].

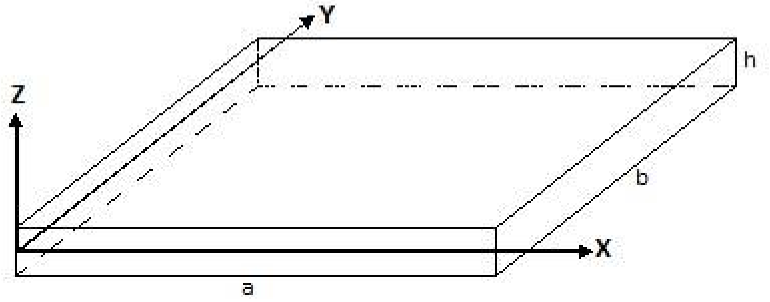

Figure 2 Geometry of the laminated composite plate

RESULT AND DISCUSSION

Convergence Study

To show the accuracy of our results we compare the results with the previous reference papers as in literature. It is clear from convergence study that there is upto 4% variation in result. The default parameters of the laminated plates are as follows

$$\frac{E_{11}}{E_{22}} = 25, \quad G_{12} = G_{13} = 0.5E_{22}, \quad G_{23} = 0.2E_{22}, \quad \nu_{12} = 0.25, \quad \rho = 2700\, kg/m^3$$

Table 1 Convergence study of non-dimensional frequencies ($\bar{\omega} = \frac{\omega b^2}{\pi^2 h}\sqrt{\frac{\rho}{E_{22}}}$) for an angle-ply laminate ($45^0/-45^0/45^0/-45^0/45^0$), ($K_s$=5/6, a/b=1, h/b=0.1) for boundary condition i.e. SSSS with respect to the results given by

Reference	Mode							
	1	2	3	4	5	6	7	8
M=N								
7	1.6227	3.1069	3.3805	3.4420	4.5588	5.1965	5.2759	6.1548
Ref .[18]	1.8608	3.3317	3.4877	3.6328	4.8878	5.4047	5.5104	6.4413
9	1.6225	3.1050	3.3784	3.4417	4.5519	5.1809	5.2611	6.1394
Ref .[18]	1.8599	3.3295	3.4873	3.6300	4.8815	5.3885	5.4956	6.4242
11	1.6252	3.10443	3.3777	3.4416	4.5505	5.1783	5.2563	6.1345
Ref .[18]	1.8593	3.3288	3.4872	3.6289	4.8793	5.3832	5.4906	6.4187
13	1.62252	3.1041	3.37750	3.4416	4.5500	5.1738	5.2543	6.1325
Ref .[18]	1.8591	3.3284	3.4872	3.6283	4.8784	5.9810	5.4885	6.4164
15	1.6225	3.1040	3.3774	3.4416	4.5498	5.1727	5.2534	6.1316
Ref .[18]	1.8588	3.3283	3.4872	3.6281	4.8779	5.3800	5.4875	6.4154

17	1.6225	3.1040	3.3773	3.4416	4.5496	5.1723	5.2530	6.1311
Ref .[18]	1.8587	3.3282	3.4872	3.6279	4.8776	5.3794	5.4870	6.4148
19	1.6225	3.1040	3.3773	3.4416	4.5496	5.1720	5.2526	6.1309
Ref .[18]	1.8586	3.3282	3.4872	3.6277	4.8775	5.3792	5.4867	6.4145
21	1.6225	3.1039	3.3773	3.4416	4.5495	5.1718	5.2525	6.1307
Ref .[18]	1.8585	3.3282	3.4872	3.6288	4.8774	5.3790	5.4865	6.4143
23	1.6225	3.1037	3.3773	3.4416	4.5495	5.1717	5.2524	6.1306
Ref .[18]	1.8584	3.3281	3.4872	3.6276	4.8773	4.3789	534864	6.4141
25	1.6225	3.1039	3.37719	3.4416	4.5495	5.1717	5.2523	6.1306
Ref .[18]	1.8584	3.3281	3.4872	3.6276	4.8773	5.3788	534863	6.4141
Ref .[6]	1.8792	3.3776	3.6924	4.9682	5.4835	5.6002	6.5475	6.9139
Ref .[9]	1.8788	3.3776	3.6921	4.9680	5.4834	5.6000	6.5473	6.9135

Table 2 Convergence study of non-dimensional frequencies ($\bar{\omega} = \frac{\omega b^2}{\pi^2 h}\sqrt{\frac{\rho}{E_{22}}}$) for an angle-ply laminate ($45^0/-45^0/45^0/-45^0/45^0$), ($K_s$=5/6, a/b=1, h/b=0.1) for boundary condition i.e. CCCC with respect to the results given by

Reference	Mode							
M=N	1	2	3	4	5	6	7	8
7	2.2051	3.6155	3.8205	5.0037	5.5057	5.6206	6.5150	6.7627
Ref .[18]	2.2483	3.6744	3.9060	5.0836	5.6111	5.7462	6.6012	6.9140
9	2.2045	3.6123	3.8173	4.9976	5.4876	5.6028	6.4963	6.7441
Ref .[18]	2.2476	3.6713	3.9026	5.0770	5.5928	5.7284	6.5821	6.8937
11	2.2042	3.6112	3.8162	4.9956	5.4817	5.5970	6.4903	6.7381
Ref .[18]	2.2474	3.6702	3.9015	5.0749	5.5867	5.7225	6.5759	6.8876
13	2.2041	3.6108	3.8158	4.9946	5.4793	5.5946	6.4878	6.7356
Ref .[18]	2.2472	3.6697	3.9010	5.0739	5.5842	5.7200	6.5733	6.8849
15	2.2040	3.6106	3.8155	4.9942	5.4782	5.5934	6.4866	6.7344
Ref .[18]	2.2482	3.6695	3.9008	5.0735	5.5830	5.7189	6.5722	6.8836
17	2.2040	3.6104	3.8153	4.9940	5.4776	5.5928	6.4860	6.7338
Ref .[18]	2.2471	3.6694	3.9007	5.0733	5.5825	5.7183	6.5715	6.8829
19	2.2040	3.6104	3.8153	4.9938	5.4773	5.5925	6.4856	6.7335
Ref .[18]	2.2470	3.6693	3.9005	5.0731	5.5822	5.7179	6.5712	6.8822
21	2.2039	3.6103	3.8152	4.9938	5.4771	5.5923	6.4854	6.7333
Ref .[18]	2.2470	3.6693	3.9005	5.0730	5.5820	5.7177	6.5710	6.8822
Ref .[6]	2.2857	3.7393	3.9813	5.1800	5.7019	5.8455	6.7167	7.0452
Ref .[15]	2.2857	3.7392	3.9813	5.1799	5.7019	5.8545	6.7166	7.0449

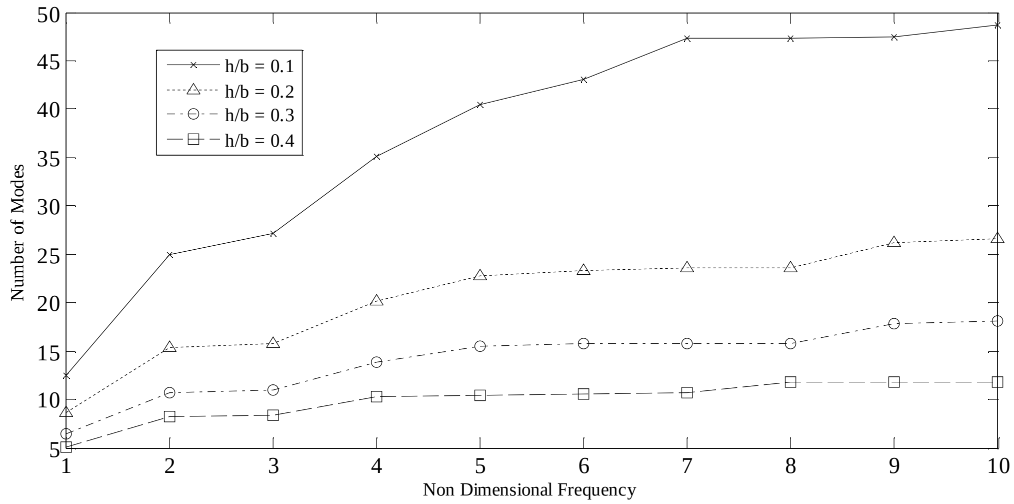

Fig-3 Variation of non dimensional frequencies ($\bar{\omega} = \frac{\omega b^2}{h}\sqrt{\frac{\rho}{E_{22}}}$) with the thickness ratio (h/b= 0.1, 0.2, 0.3, 0.4) of plate for symmetric laminate (90°/0°/60°/-60°/0°/0°/-60°/60°/0°/90°) for SSSS external condition (a/b=1)

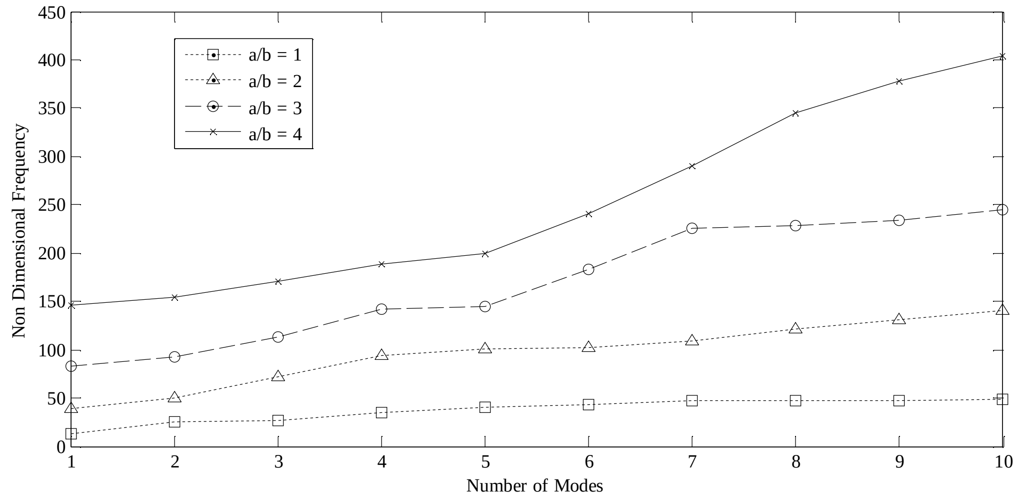

Fig-4 Variation of non dimensional ($\bar{\omega} = \frac{\omega b^2}{h}\sqrt{\frac{\rho}{E_{22}}}$) frequencies with the aspect ratio (a/b= 1, 2, 3, 4) of plate for symmetric laminate (90°/0°/60°/-60°/0°/0°/-60°/60°/0°/90°) for SSSS external condition (h/b=0.1)

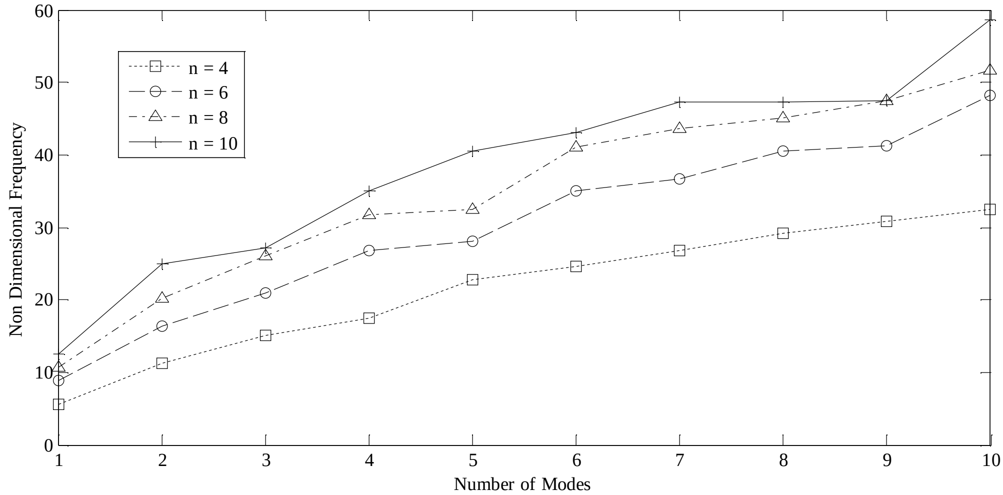

Fig-5 Variation of non dimensional frequencies ($\bar{\omega} = \frac{\omega b^2}{h}\sqrt{\frac{\rho}{E_{22}}}$) with the number of layers (n= 4, 6, 8, 10) of plate for symmetric laminate (90°/0°/60°/-60°/0°/0°/-60°/60°/0°/90°) for SSSS external condition (a/b=1, h/b=0.1)

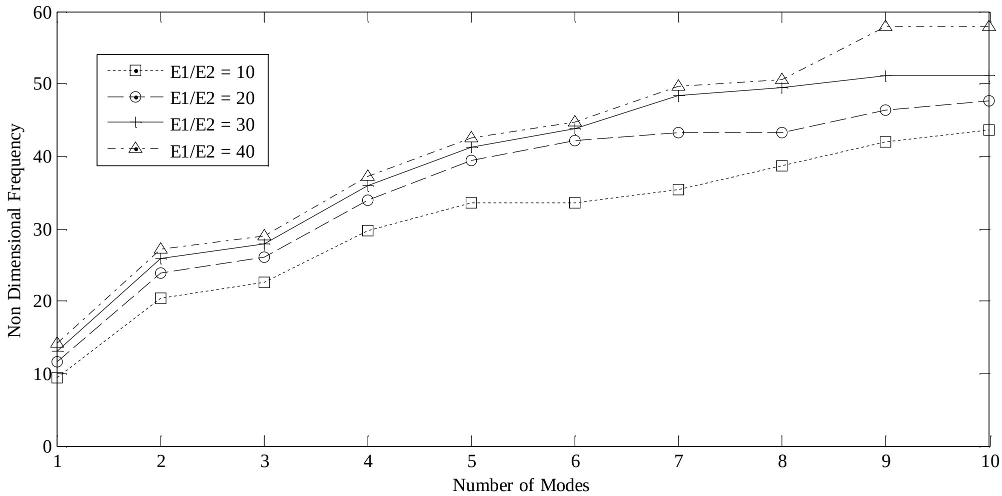

Fig-6 Variation of non dimensional frequencies ($\bar{\omega} = \frac{\omega b^2}{h}\sqrt{\frac{\rho}{E_{22}}}$) with the material properties (E11=10,20,30,40) of plate for symmetric laminate (90°/0°/60°/-60°/0°/0°/-60°/60°/0°/90°) for SSSS external condition (a/b=1, h/b=0.1)

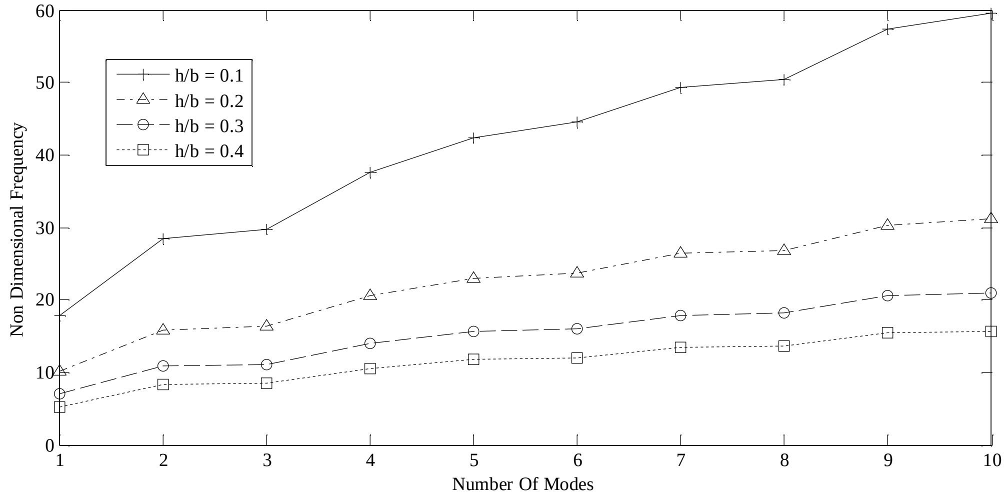

Fig-7 Variation of non dimensional frequencies ($\overline{\omega} = \frac{\omega b^2}{h}\sqrt{\frac{\rho}{E_{22}}}$) with the thickness ratio (h/b= 0.1, 0.2, 0.3, 0.4) of plate for symmetric laminate (90°/0°/60°/-60°/0°/0°/-60°/60°/0°/90°) for CCCC external condition (a/b=1)

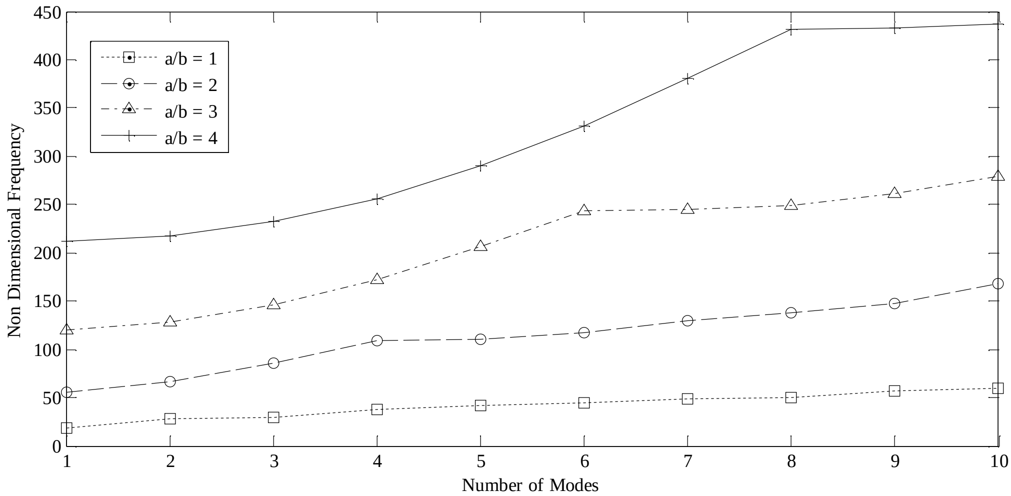

Fig-8 Variation of non dimensional ($\overline{\omega} = \frac{\omega b^2}{h}\sqrt{\frac{\rho}{E_{22}}}$) frequencies with the aspect ratio (a/b= 1, 2, 3, 4) of plate for symmetric laminate (90°/0°/60°/-60°/0°/0°/-60°/60°/0°/90°) for CCCC external condition (h/b=0.1)

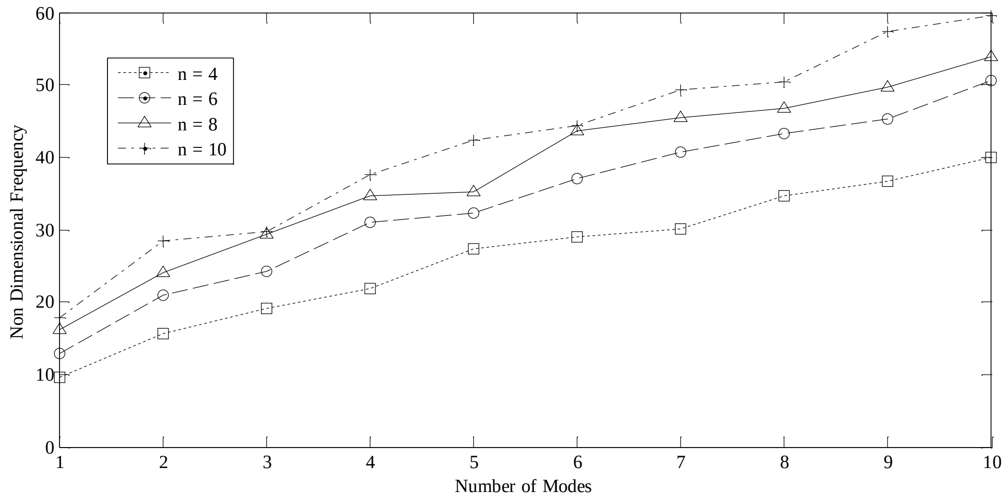

Fig-9 Variation of non dimensional frequencies ($\bar{\omega} = \frac{\omega b^2}{h}\sqrt{\frac{\rho}{E_{22}}}$) with the number of layers (n= 4, 6, 8, 10) of plate for symmetric laminate (90°/0°/60°/-60°/0°/0°/-60°/60°/0°/90°) for CCCC external condition (a/b=1, h/b=0.1)

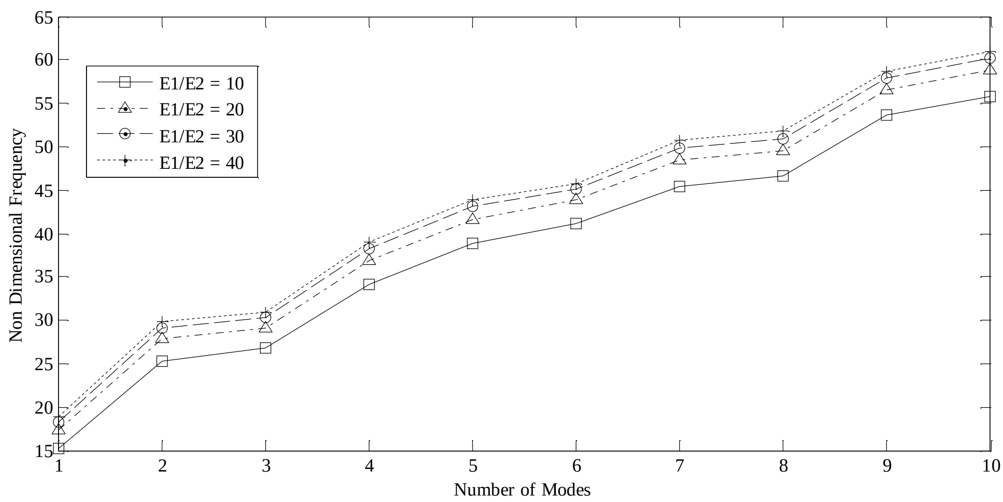

Fig-10 Variation of non dimensional frequencies ($\bar{\omega} = \frac{\omega b^2}{h}\sqrt{\frac{\rho}{E_{22}}}$) with the material properties (E11=10,20,30,40) of plate for symmetric laminate (90°/0°/60°/-60°/0°/0°/-60°/60°/0°/90°) for CCCC external condition (a/b=1, h/b=0.1)

CONCLUSION

1. The variation of non-dimensional fundamental frequency decrease as h/b increases.
2. Non-dimensional fundamental frequency of vibration is found to be increases with increasing aspect ratio in different boundary conditions.
3. The result shows that non dimensional Frequency increases as number of layer increases in different boundary conditions.
4. The variation of non-dimensional fundamental frequency increase as E_{11}/E_{22} increases.
5. The boundary conditions of the plate play an important role in the frequency of vibration of the system.

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